The Stellar Seismic Indices (SSI) data base : D 3.6 Report: Use of seismic scaling relations for deriving stellar mass, radius and surface gravity.

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1 Introduction

Thanks to CoRoT (CNES and European's parterns) and Kepler (NASA) seismic observations, it is now possible to detect many solar-like oscillations in a very large number of stars. These oscillation spectra show characteristic patterns, which can be simply characterized using so-called *stellar seismic indices*. More specifically, the stellar seismic indices correspond to some characteristic global seismic numbers,which are extracted from the oscillation spectra of solar-like pulsating stars. Two main indices are considered here:

- the maximum peak frequency, $\nu_{\max}$: this is in general the frequency at which the oscillation spectrum peaks in the power density spectrum (PDS) ;
- the mean large separation, $\Delta\nu$: this quantity corresponds to the mean frequency spacing between two consecutive p-modes (with the same angular degree) ;

These two seismic indices obey characteristic scaling relations that depend directly on the radius, mass and effective temperature of the star. From the knowledge of the effective temperature and the measure of these two seismic indices, it is possible to estimate the mass and radius of the star. Following our plans, these two indices are now provided by an automatic procedure (developed by Raphael Peralta during his PhD thesis at Paris Observatory) and made available at the SSI data base.

Because they are relatively easy to measure thanks to CoRoT and *Kepler* it has already been possible to infer seismic indices in about 10 000 red giants and sub-giants, and subsequently to derive their mass, radius and evolutionary status. Theses seismic indices are more and more used in the physics but also for the study of Galactic populations (see the reviews by Belkacem 2012; Belkacem et al. 2013). These seimic indices have opened the way toward what we name now ensemble asteroseismology.

By June 2016, the Stellar Seismic Indices (SSI) data base will provide such sets of seismic indices extracted in an *automatic* and *homogeneous* way for a large set of CoRoT and *Kepler* targets. We expect to feed this data base with seismic indices extracted from about 12 000 *Kepler* red giant light-curves and from about 5 000 *CoRoT* red giant light-curves. This data base will be soon directly accessible from the SSI website (`http://ssi.lesia.obspm.fr/`). It will also be accessible from the Seismic Plus Portal (`http://voparis-spaceinn.obspm.fr/seismic-plus/`). It is planned to implement in this portal a tool that will allow any user to derive the mass, radius and surface gravity of a star for which the seismic indices $\nu_{\max}$ and $\Delta\nu$ together with the effective temperature T_{eff} are available, either in the SSI data base or any other seismic data base accessible from the portal. This document presents the scaling relations that will be implemented in this tool to derive these stellar parameters and the associated (internal) errors. Note that only the internal errors due to the seismic analysis are taken into account ; the other sources of uncertainties are discussed in e.g. Belkacem (2012) and Belkacem et al. (2013).

2 The scaling relations

By definition, the scaling relations denote the proportionality between a given seismic index and various stellar parameters. We recall here the way these scaling relations are derived.

2.1 The seismic index $\nu_{\max}$

Brown et al. (1991) have suggested that $\nu_{\max}$ is directly proportional to the cut-off frequency ν_c of the stellar atmosphere (e.g. Kjeldsen & Bedding 1995). We recall that ν_c corresponds to the frequency above which an acoustic wave reflexion at the stellar surfaceis no longer total. Accordingly, $\nu_{\max}$ scales as

$$\nu_{\max} \propto \nu_c \,. \tag{1}$$

For an isothermal atmosphere, the cut off frequency ν_c can be approximately expressed as (Balmforth & Gough 1990; Stello et al. 2009)

$$\nu_c = \frac{c_s}{2H_p} \,, \tag{2}$$

where c_s is the sound speed and H_p the pressure scale-height.

Following Kjeldsen & Bedding (1995) the sound speed is proportional to the effective temperature T_{eff} as:

$$c_s \propto \sqrt{T_{\text{eff}}} \,. \tag{3}$$

Now, from the hydrostatic equation ($P = \rho g H_p$), the equation of ideal gas ($P \propto \rho T$), the definition of the surface gravity g ($g \propto M/R^2$), and finally the Stefan-Boltzmann's equation ($L \propto R^2 T_{\text{eff}}^4$) one can write:

$$H_p \propto \frac{T_{\text{eff}}}{g} \propto \frac{T_{\text{eff}} R^2}{M} \propto \frac{L}{M T_{\text{eff}}^3} \,, \tag{4}$$

where g is the surface gravity, L the luminosity, R the radius and M the mass. We finally derive the scaling relation for $\nu_{\max}$, that is:

$$\nu_{\max} \propto \nu_c \propto \frac{g}{\sqrt{T_{\text{eff}}}} \propto \frac{M}{R^2 \sqrt{T_{\text{eff}}}} \,. \tag{5}$$

2.2 The seismic index $\Delta\nu$

Ulrich (1986) first showed that the mean large separation, $\Delta\nu$, is directly linked to the mean stellar density:

$$\Delta\nu \propto \overline{\rho}^{\;1/2} \,. \tag{6}$$

By definition, the mean stellar density, $\overline{\rho}$, is related to the star mass M and radius R, that is $\rho = M/V \propto M/R^3$. We then establish the scaling relation which $\Delta\nu$ obeys:

$$\Delta\nu \propto \left(\frac{M}{R^3}\right)^{1/2} \propto \left(\frac{g}{R}\right)^{1/2} \,. \tag{7}$$

Parameter	$\delta(err)$ CoRoT (%)	$\delta(err)$ *Kepler* (%)
$\nu_{\rm max}$	2%	-0.5%
$\Delta\nu$	0.1%	-0.03%
$M/\mathrm{M}_\odot$	—	-6%
$R/\mathrm{R}_\odot$	—	-2%

Table 1: Median values of the relative internal errors associated with the various parameters derived from the analysis of CoRoT and *Kepler* light-curves.

3 The stellar parameters M, R and $\log g$

By combining Eq. (5) and Eq. (7), one derives the scaling relations for M and R:

$$\frac{R}{R_\odot} = \left(\frac{\nu_{\rm max}}{\nu_{\rm ref}}\right)\left(\frac{\Delta\nu}{\Delta\nu_{\rm ref}}\right)^{-2}\left(\frac{T_{\rm eff}}{T_\odot}\right)^{1/2} \tag{8}$$

$$\frac{M}{M_\odot} = \left(\frac{\nu_{\rm max}}{\nu_{\rm ref}}\right)^{3}\left(\frac{\Delta\nu}{\Delta\nu_{\rm ref}}\right)^{-4}\left(\frac{T_{\rm eff}}{T_\odot}\right)^{3/2} . \tag{9}$$

These scaling relations are normalised with respect to the reference values introduced by Mosser et al. (2013): $\Delta\nu_{\rm ref} = 138.8$ μHz, $\nu_{\rm ref} = 3104$ μHz and $T_\odot = 5777$ K.

Concerning the logarithm of the surface gravity, $\log g$, given the fact that $\nu_{\rm max} \propto g\, T_{\rm eff}^{-1/2}$ (see Eq. 5), one has:

$$\log g = \log g_\odot + \log\left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right) + \frac{1}{2}\log\left(\frac{T_{\rm eff}}{\mathrm{T}_{\rm eff,\odot}}\right) , \tag{10}$$

where $\log g_\odot = 4.438$ [cm/s^2], $\nu_{\rm max,\odot} = 3104$ μHz and $\mathrm{T}_{\rm eff,\odot} = 5777$ K. Note that log stands here for the decimal logarithm.

The uncertainties associated with the parameters M, R and $\log g$ are respectively given by the following equations:

$$\delta M = M\sqrt{\left(\frac{3\delta\nu_{\rm max}}{\nu_{\rm max}}\right)^2 + \left(\frac{4\delta\Delta\nu}{\Delta\nu}\right)^2 + \left(\frac{3}{2}\frac{\delta T_{\rm eff}}{T_{\rm eff}}\right)^2} \tag{11}$$

$$\delta R = R\sqrt{\left(\frac{\delta\nu_{\rm max}}{\nu_{\rm max}}\right)^2 + \left(\frac{2\delta\Delta\nu}{\Delta\nu}\right)^2 + \left(\frac{1}{2}\frac{\delta T_{\rm eff}}{T_{\rm eff}}\right)^2} \tag{12}$$

$$\delta \log g = \frac{1}{\ln 10}\sqrt{\left(\frac{\delta\nu_{\rm max}}{\nu_{\rm max}}\right)^2 + \left(\frac{1}{2}\frac{\delta T_{\rm eff}}{T_{\rm eff}}\right)^2} \tag{13}$$

where $\delta T_{\rm eff}$ is the error associated with $T_{\rm eff}$ while $\delta\nu_{\rm max}$, and $\delta\Delta\nu$ are respectively the (internal) errors associated with the seismic parameters $\nu_{\rm max}$ and $\Delta\nu$. Note that the estimate of the mass is more sensitive to the errors related to the seismic parameters and $T_{\rm eff}$ than is the radius. We also note that $\log g$ is weakly sensitive to $T_{\rm eff}$. Indeed, an error of $\delta T_{\rm eff} = 100$ K results in an error on $\log g$ equal to $\delta \log g = 0.005$ dex. Typical values of the internal errors associated with the seismic and stellar parameters are reported in Table 1.

To illustrate our ability to derive the stellar parameters for a large number of objects, we have extracted $\Delta\nu$ and $\nu_{\rm max}$ for about 12 000 *Kepler* red giants using the extraction pipeline associated with the SSI data base (Peralta et al 2016, in prep.). Concerning $T_{\rm eff}$, Huber et al. (2014) have presented a compilation of literature values derived from different observational techniques (photometry, spectroscopy, and exoplanet transits). Using now the scaling relations associated with $\nu_{\rm max}$ (Eq. 5) and $\Delta\nu$ (Eq. 7), we estimate the mass and radius for 11 937 *Kepler* red giants. These values are presented in Fig. 1. Stars belonging to the clump and Red Giant Branch (RGB) are identified following Vrard et al (2015, submitted).

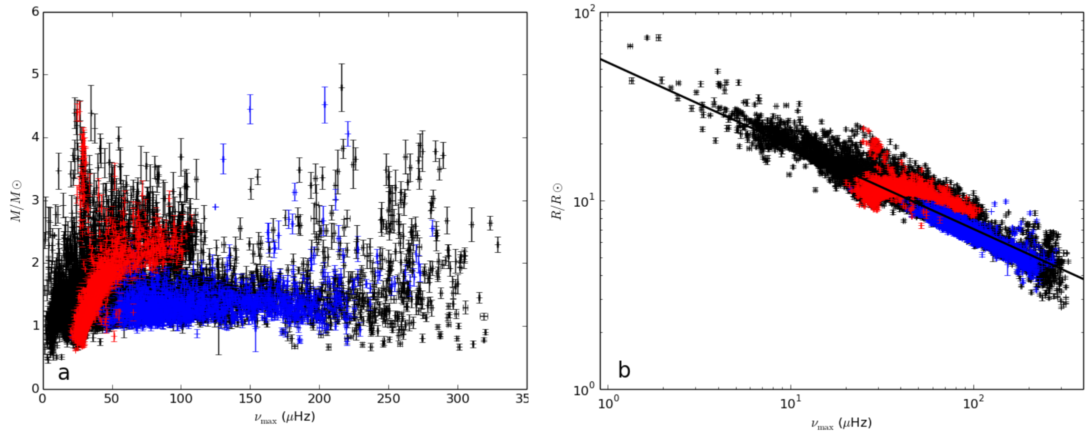

Figure 1: Distribution of the stellar masse (a) and radius (b) as a function of $\nu_{\max}$, derived for 11 937 *Kepler* red giants. The stars identified as belonging respectively to the clump and the Red Giant Branch (RGB) are presented in red and blue. The solid black line corresponds to the adjusted scaling relations $R/R_\odot = (59.3 \pm 0.4)\ \nu_{\max}^{-0.464\pm0.002}$.

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